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ANALYSIS OF LATERALLY LOADED FIXED HEADED SINGLE FLOATING PILE IN MULTILAYERED SOIL USING BEF APPROACH

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ABSTRACT: Structures resting on pile foundation are generally subjected to lateral loads and moments acting on the pile head in addition to vertical loads. Horizontal forces originate due to wind, traffic, earth pressure, water wave and seismic forces or their combination. Analysis and design of piles subjected to lateral forces and moments is very important for ensuring the stability and serviceability of structure. In designing laterally loaded piles, the pile head deflection is very important which depends on soil type, pile installation, pile flexibility (or pile stiffness), loading condition and type of fixity of pile with pile cap. The flexural behaviour of a pile is a function of the interaction between the soil and the pile and is governed by the properties of both. In the present study, lateral load behavior of single piles in layered soils i.e. alternate layer of clay and sand is conducted, for a range of subgrade modulus representing various soil types. The analysis is carried out considering fixed headed pile and floating tip at the base. Since soil response is complex hence idealized models are used for the analysis. The analysis has been carried out with the aid of Finite Difference Method (FDM) incorporated in the software Beams on Elastic Foundations (BEF). The analysis helps to determine the lateral pile capacity, moment and length of fixity for different conditions of pile diameter and embedment through varieties of stratified layers.

Keywords: Laterally loaded pile, Fixed-headed pile, BEF, Linear analysis

1. INTRODUCTION

Pile foundations are used in important structures like transmission line towers, bridges, chimney, high rise building, overhead water tank, offshore structures like oil platform, jetties etc that are subjected to large lateral forces and moments in addition to the vertical load. Lateral force arises due to wind, earthquake forces, wave action, traffic, lateral earth pressure and their combinations. To ensure the stability and serviceability of various structures, it is essential to determine the allowable lateral load corresponding to an acceptable lateral deflection and corresponding bending moment at the top of pile i.e. pile head. The analysis of laterally loaded pile is a very complex problem since flexural behaviour of pile is a function of soil structure interaction. Both the soil properties and pile element properties need to be considered carefully when analyzing the behaviour of such a system.

Numerical research studies, both theoretical and experimental, performed by several investigators on laterally loaded pile to determine their ultimate lateral load and displacements are available in the literature. Several researchers have given different approaches for solving the problem [1,2,3,4,5,6,7,8,9,10]. With the help of charts and tables given in [11], subgrade modulus and lateral pile capacity can be easily determined. However, it is applicable only for a single layered soil i.e. either sand or clay with constant subgrade reaction. For multilayered soil, no such provision is available in the code and one has to go for rigorous methods for the same.

The present article reports the findings of a study to estimate the lateral load capacity of a single pile embedded in layered soil deposit, based on the permissible deflection of 1% of the pile diameter at the bottom of pile cap. Pile are considered to be fixed headed i.e. rigid pile cap is present at top thus

allowing no rotation at top. Floating pile is considered at bottom. Flexural response of the pile (in terms of deflection, bending moment, shear force and contact stress profiles) has been illustrated using Beams on Elastic Foundations (BEF) utilizing Finite Difference Method (FDM) Based on the results, the article reports the comparison of lateral pile capacity, moment and length of fixity considering for different pile diameters.

2. LOAD TRANSFER MECHANISM OF LATERALLY LOADED PILE

When external horizontal load acts on the pile head, piles behave as transversely loaded beam i.e. a part of the pile tries to shift horizontally in the direction of the applied load causing bending, rotation/ translation of pile. The pile presses against the soil lying in the direction of applied load and generates compressive and shear stresses in soil. To satisfy equilibrium, soil must provide a resistance along the entire pile shaft to balance the external horizontal force and moment. At every point along the length of the pile, relationship generally exists between soil resistance (p) and pile deflection (y) which may be linear or non-linear. When linear relation is assumed (i.e. $p=ky$), k is called as the horizontal subgrade modulus. Figure 1 exhibits a typical deflected shape of the pile and the associated soil reaction curve.

3. METHOD OF ANALYSIS

Beams on Elastic Foundation (BEF) approach utilizes the concept of beam resting on elastic Winkler foundation rotated by 90 degrees. Here the beam represents the piles and the foundation represents the passive soil mass. In this condition, the pile is considered to be supported by a series of discrete elastic springs so that the compression (or extension) of the spring (which is same as deflection of ground) is proportional to the applied load (Fig. 2). The continuous nature of the soil medium is ignored in this analysis. The spring constant represents the stiffness of the ground against the applied load. The soil stiffness parameter k is taken as lateral subgrade modulus of soil. In order to simplify the problem, the springs are assumed to be linear elastic up to a certain value of pile deflection and perfectly plastic beyond that value. However, more correctly, they should be modeled as nonlinear springs.

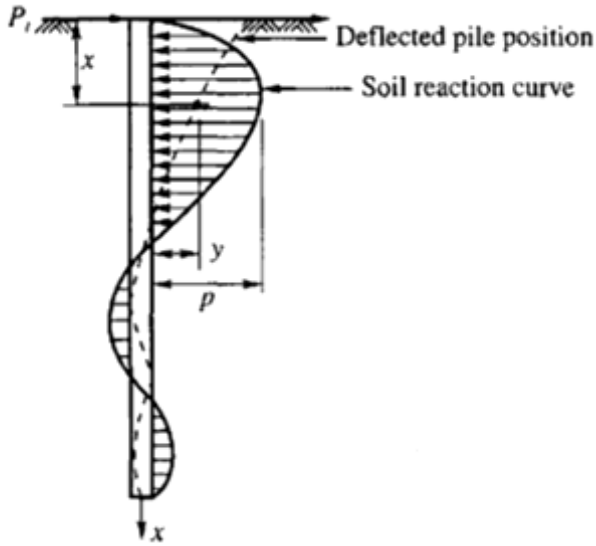

Fig 1a. Typical representation of the deflection of a laterally loaded pile and the associated soil reaction curve

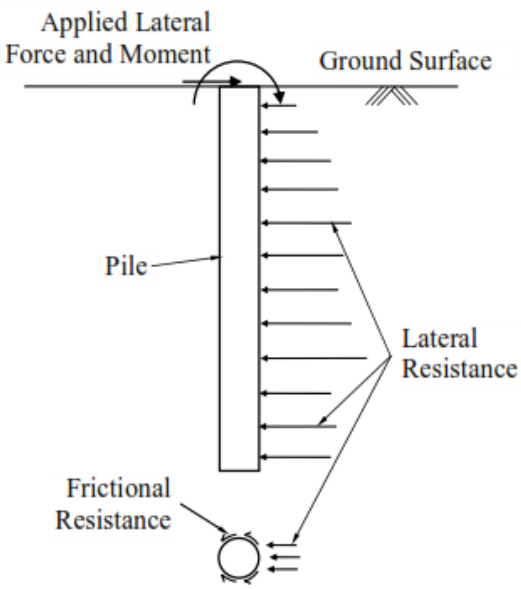

Fig 1b. Generation of lateral and shear stresses around a laterally loaded pile

In foundation springs, the spring constant multiplied by the spring deflection (which is the same as the beam deflection), produces the resistive force of the foundation (ground) per unit beam length. This method is also called subgrade reaction approach. A

fourth order linear differential equation governing the beam deflection for such a beam-foundation system is used. The input parameters required are the elastic modulus and geometry of the beam, the spring constant of the foundation (soil) and the magnitude and distribution of the applied load. Fig. 3 shows a arbitrarily loaded prismatic beam resting on Winkler foundation.

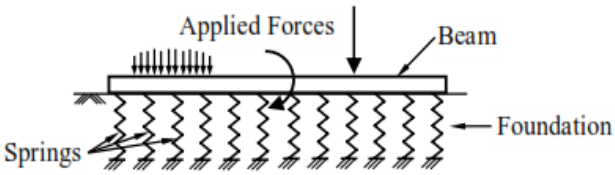

Fig 2. Typical representation of laterally loaded piles through BEF analysis

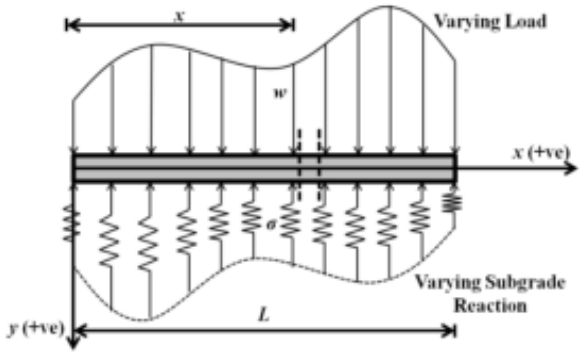

Fig. 3 Arbitrarily loaded prismatic beam resting on Winkler foundation

The governing deflection response of the above beam is expressed as:

$$EI\frac{\partial^4 y}{\partial x^4} = -M = w - ky \qquad (1)$$

where, E is the modulus of elasticity and I is the moment of inertia of the beam, y is the deflection at any point of the beam, w is the magnitude of applied load, and k is the modulus of subgrade reaction of the elastic foundation. Beam deflection, bending moment and shear force along the span of the beam can be determined easily after solving the equation.

In order to solve the above problem numerically by finite difference method, the beam is discretized into several nodes, and the central difference scheme of the finite difference theory is utilized to express the governing deflection response at any node 'i' as:

$$\left[y_{i-2} - 4y_{i-1} + (6 + \frac{k_i h^4}{EI}) y_i - 4y_{i+2} + y_{i+2} \right] = \frac{P_i h^3}{EI} \qquad (2)$$

where, P_i is the computed equivalent nodal load, and h is the uniform spacing of two nodes. The above expression is further modified for appropriate boundary conditions and changes in the flexural rigidity of the beam and subgrade modulus of the soil.

4. PROBLEM STATEMENT

Fig. 4 depicts a fixed-headed single floating pile embedded in a stratified soil. The four problems described herein are supported with the following basic information:

- Pile diameter = 1000mm, 1200mm, 1500 mm
- Bottom of Pile cap i.e. pile head depth = 2.5m from existing ground level (EGL)
- Pile head is fixed
- Concrete grade is M25
- Deflection at bottom of pile cap i.e. at pile head is 1% of pile diameter.

The determination of the input stiffness and subgrade parameters are illustrated in [12]

As stated above, the method of analysis used for the present study is Beams on Elastic foundation (BEF)

The results are expressed in terms of

- Lateral load capacity of the pile and the effect of pile diameter as obtained from the load-deformation curves
- Moment at pile head
- Length of fixity of pile from bottom of pile cap

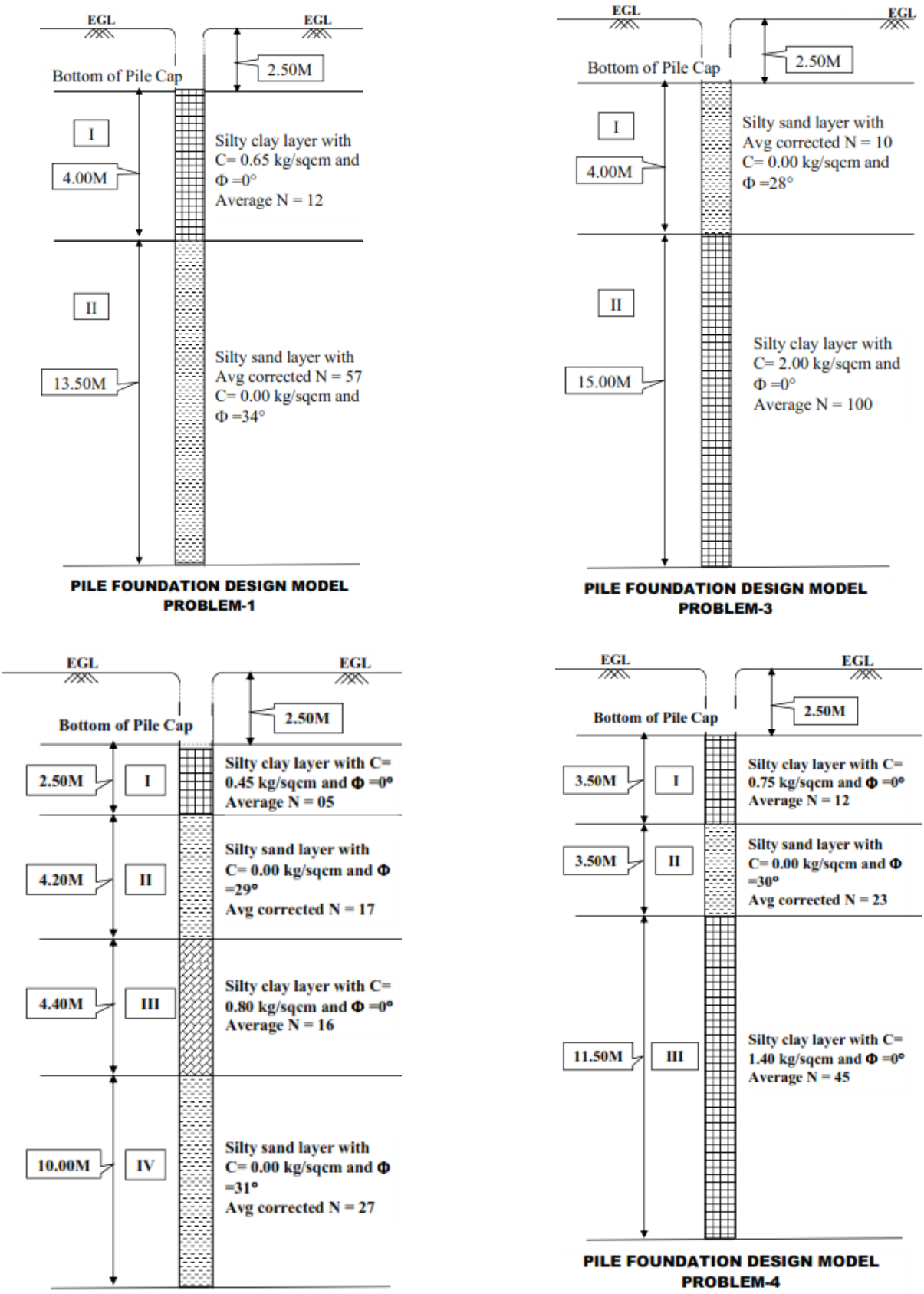

Fig. 4 Soil-pile configurations of the present study

- Deflection, slope, bending moment, shear force and contact stress distribution diagrams

5. BEF BASED LINEAR-ELASTIC ANALYSIS

A linear elastic analysis has been carried out using BEF, considering the laterally loaded pile as a beam resting on elastic foundation (representing the soil stratification with varying subgrade modulus). A vertical load is applied at the tip of the free-headed pile. The bottom of the pile is considered to be restrained both from translation and rotation. BEF does not account for water table in the analysis.

5.1. Estimation of Model Parameters

For an analysis in BEF, the contributory parameters are (a) Pile - the modulus of elasticity of pile material, geometry configuration (length and cross-section dimensions) and moment of inertia, and (b) Soil - modulus of elasticity and subgrade modulus of soil.

Depending on the grade of concrete, the elastic modulus of concrete (E_s) is estimated (using Indian Standards [11]) as:

$$E_c = 5000\sqrt{f_{ck}} = 25 \text{ kN/mm}^2 \quad (3)$$

where, f_{ck} is the grade of concrete (in kN/mm^2)

Based on standard literature [6], the modulus of the clayey and sandy stratum is estimated as:

$$E_{s(clay)} = 350c = 227.5 \text{ kg/cm}^2 \quad (4)$$

$$E_{s(sand)} = 400 + (10.5 \times N) = 998.5 \text{ kg/cm}^2 \quad (5)$$

where, c is the cohesion of clayey soil, and N is the SPT blow count obtained in a sandy soil. Considering a 1000 mm diameter of pile, the moment of inertia is estimated to be 4.9x10^6 cm^4. Using Vesic's expression [2], the modulus of subgrade reaction (k_s) (Pile diameter = 1000 mm) is estimated to be 11.76 and 54.88 kN/m^2/mm respectively for silty-clayey and sandy strata as shown in Fig. 3.

$$k_S = \frac{0.65E_S}{D\left(1-\nu_S^2\right)}\sqrt[12]{\frac{E_S D^4}{E_C I_p}} \quad (6)$$

where, D is the pile diameter, E_s is modulus of elasticity of soil, υ_s is the Poisson's ratio of soil (0.35 for clayey soil and 0.25 for sandy soil), and I_p is moment of inertia of the pile. For piles of other diameters, the subgrade modulus has been similarly calculated.

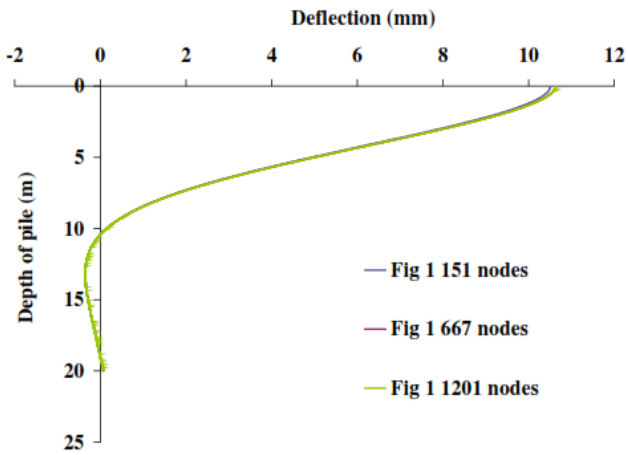

Fig. 5 Convergence of deflection with the variation in the number of nodes

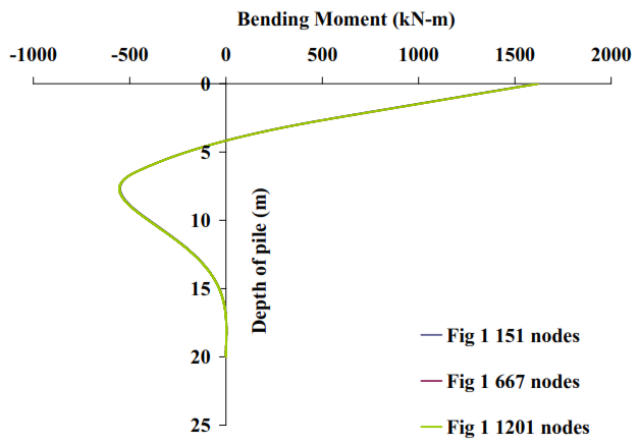

Fig. 6 Convergence of bending moment with the variation in the number of nodes

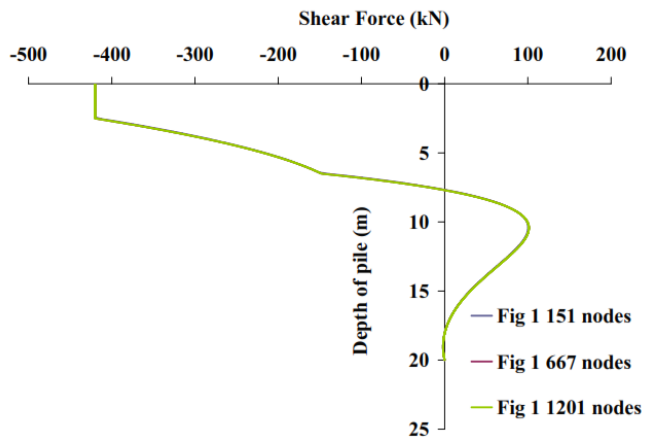

Fig. 7 Convergence of shear force with the variation in the number of nodes

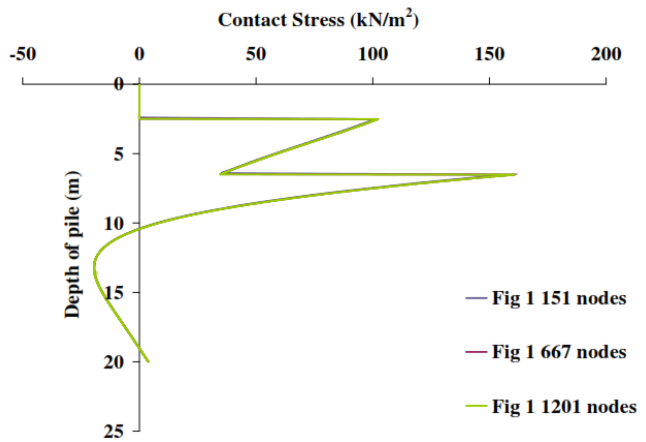

Fig. 8 Convergence of contact stress with the variation in the number of nodes

5.2. Mesh Discretization and Convergence Study

Finite difference method is subjected to various types of numerical instabilities originating due to either sparse meshing (missing response variations) or even very dense meshing (progressive error accumulation). Both the instabilities result in a non-convergent solution. Hence, it is necessary to carry out a sensitivity study and arrive at an optimum number of nodes which results in a stable and convergent solution which does not vary with small changes in the number of nodes [13]. In BEF, the sensitivity study was carried with 151, 677 and 1201 nodes and the changes in the flexural responses were investigated. 151 nodes were considered to be optimum and have been used in further studies. Fig. 5-8 depicts the convergence of the flexural responses of the pile with the variation of the number of nodes.

5.4. Load-Deformation Response and Lateral Load Capacity of Pile

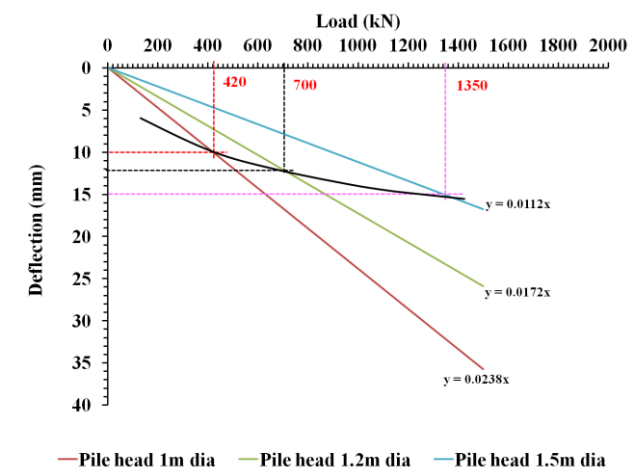

Fig. 9 Load-deformation and lateral load capacity of pile using BEF – Problem 1

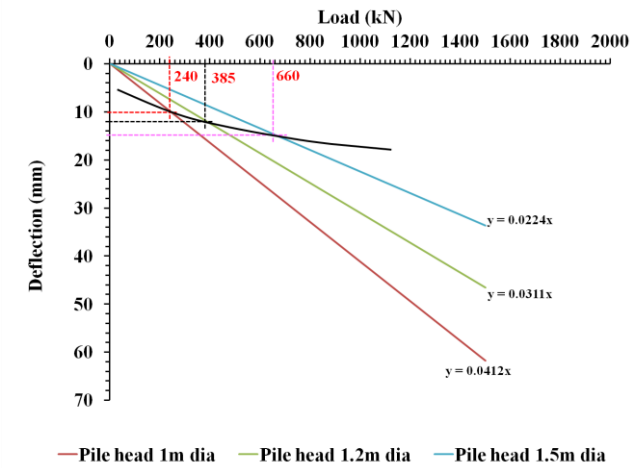

Fig. 10 Load-deformation and lateral load capacity of pile using BEF – Problem 2

The lateral load capacity of the fixed-headed pile is estimated based on the maximum permissible deformation criterion of 1% of the pile diameter at its cut-off level. For the present study, the load at the pile head is continually increased till the permissible deflection at the pile head or cut-off level is reached. Fig. 9-12 depicts the load-deflection response of the pile. Since BEF operates

on linear elastic springs, the response curves have also been obtained as linear. It is observable that the lateral load capacity of pile increases with the diameter of the pile due to the enhanced flexural stiffness of the pile.

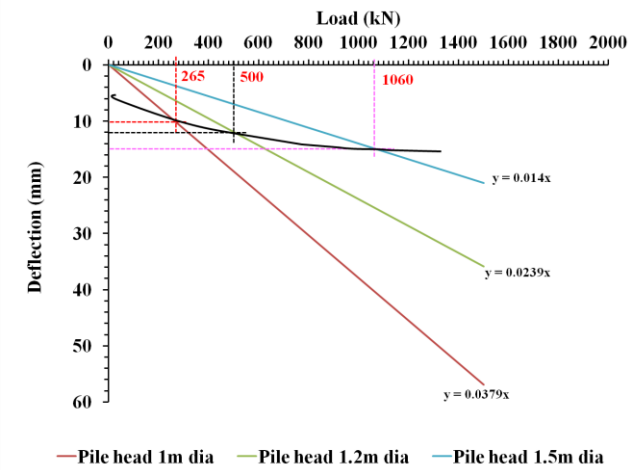

Fig. 11 Load-deformation and lateral load capacity of pile using BEF – Problem 3

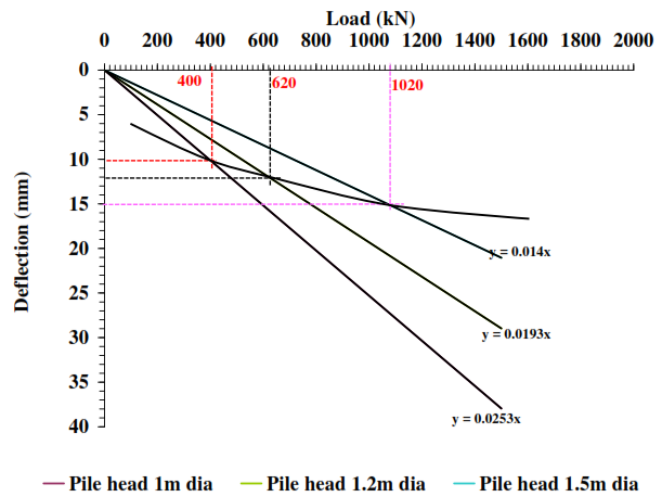

Fig. 12 Load-deformation and lateral load capacity of pile using BEF – Problem 4

The lateral load capacity of the pile, as estimated above, is further used to determine the flexural response and estimate the maximum bending moment, shear force and contact stress generated at the verge of failure. Fig. 13 depicts such a typical flexural response as obtained from the software. In actual practice, the pile needs to be designed based on these maximum magnitudes. Moreover, the length of fixity of the laterally-loaded pile has also been determined from the maximum moment/zero shear force criterion. The point of fixity is referred to as that point above which the pile can be modelled and analyzed a fixed cantilever beam. Beneath this point, the pile is considered to be fixed and does not contribute to rotation of the pile. Table 1 enlists the response of the pile (for various diameters) at its lateral load capacity. The table reflects that increase in pile diameter results in increment of the lateral load capacity as well as the length of fixity of the same.

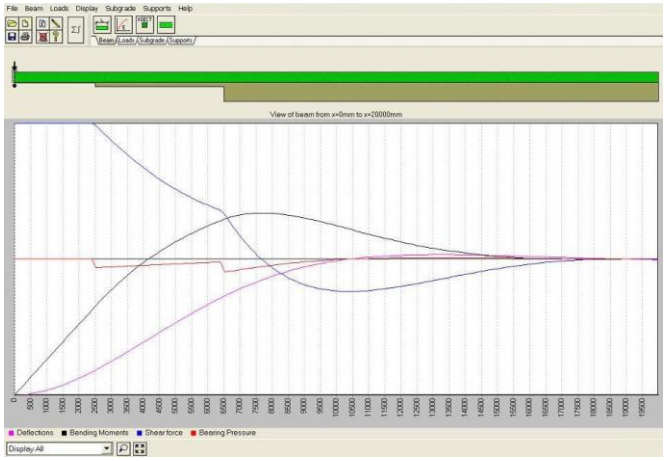

Fig. 13 Typical flexural response of fixed-headed single vertical pile as obtained from BEF analysis

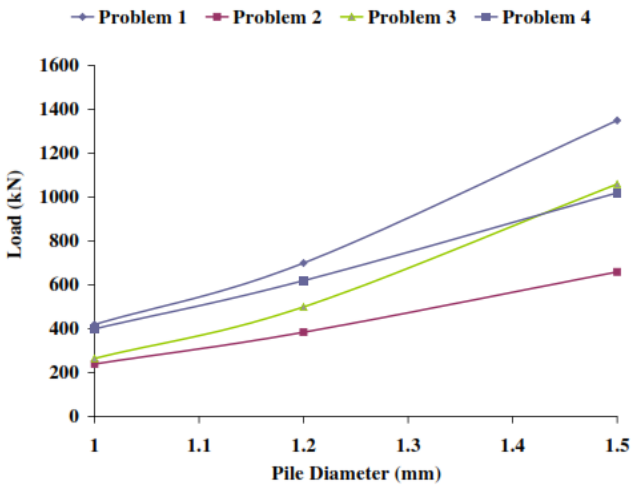

Fig. 14 Effect of diameter on the lateral load capacity of piles

5.5. Effects of Pile Diameter on Lateral Load Capacity

It is observed that the increase in diameter results in the increase in the lateral load capacity of the pile. This is accounted due to the enhanced flexural rigidity of the pile which lessens the magnitude of deflection under the action of same magnitude of loading. Figure 14 depicts the effect of the diameter in the enhancement of lateral load capacity, the results being conglomerated for all the pile-soil configurations mentioned earlier.

5.6. Flexural Response of Pile at Lateral Load Capacity

Once the lateral load capacity of the pile has been estimated using the methodology stated earlier, the flexural response of the pile has been investigated to determine the following: (a) Maximum bending moment, shear force and contact stress in the pile (b) Moment generated at the cut-off level, and (c) the point of fixity of the pile. The point of fixity of a laterally loaded pile is defined as the point of maximum bending moment (and hence, the point of zero shear force) for the pile. It is also referred as the point above which the pile can be modeled and analyzed as a fixed cantilever beam. Beneath this point, the pile is considered fixed and does not contribute to the rotation of the pile. Fig. 15-18 depicts the flexural responses for the pile of different diameters embedded in the different soil stratifications as mentioned earlier.

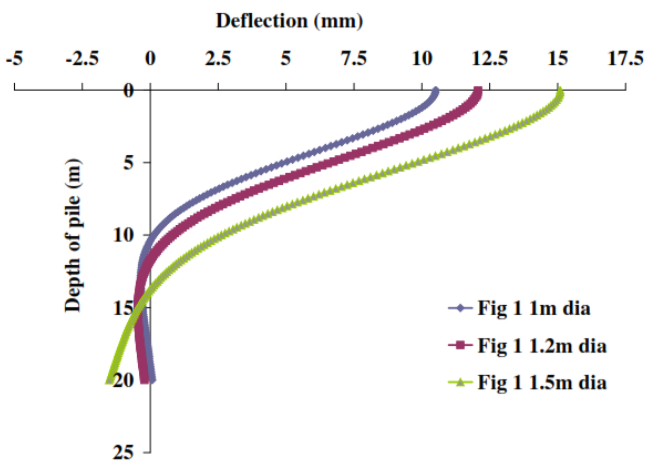

Fig 15a Effect of pile diameter on the deflection profile of fixed headed pile (Problem 1)

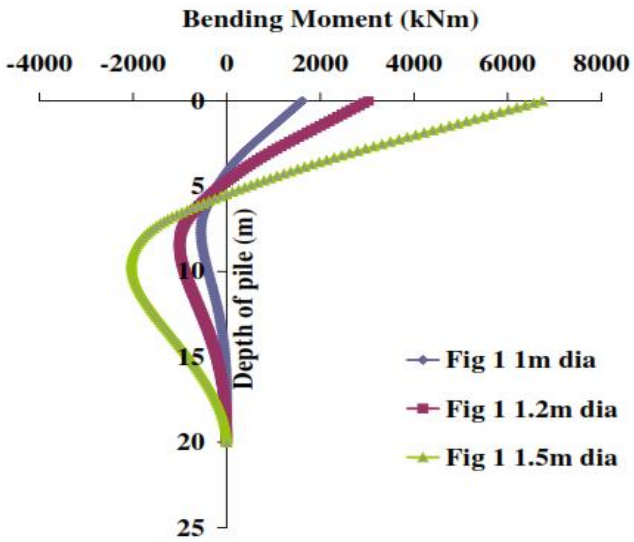

Fig 15b Effect of pile diameter on the bending moment profile of fixed headed pile (Problem 1)

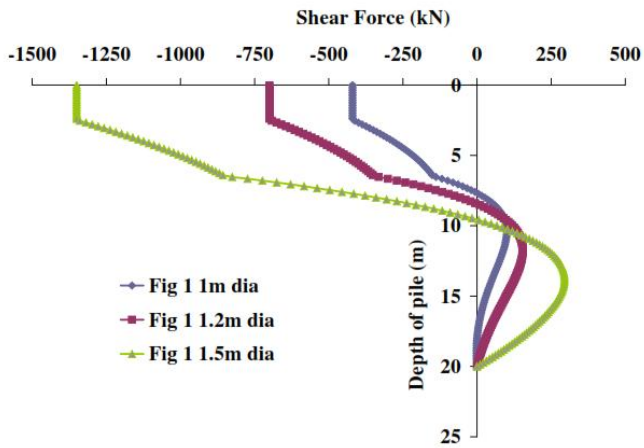

Fig 15c Effect of pile diameter on the shear force profile of fixed headed pile (Problem 1)

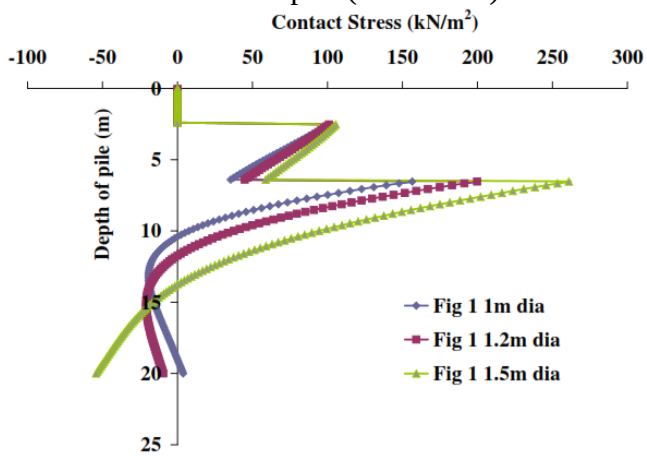

Fig 15d Effect of pile diameter on the contact stress profile of fixed headed pile (Problem 1)

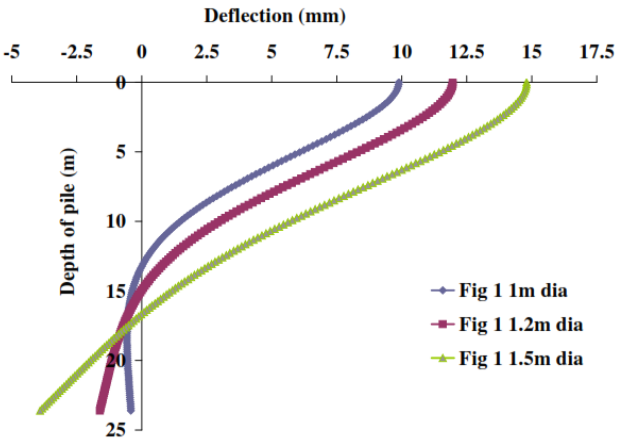

Fig 16a Effect of pile diameter on the deflection profile of fixed headed pile (Problem 2)

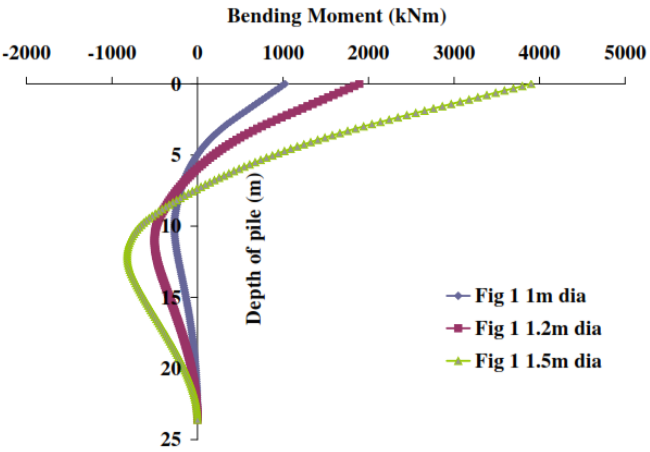

Fig 16b Effect of pile diameter on the bending moment profile of fixed headed pile (Problem 2)

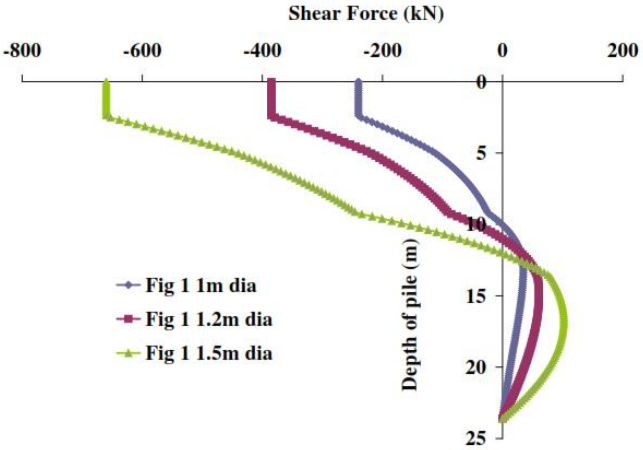

Fig 16c Effect of pile diameter on the shear force profile of fixed headed pile (Problem 2)

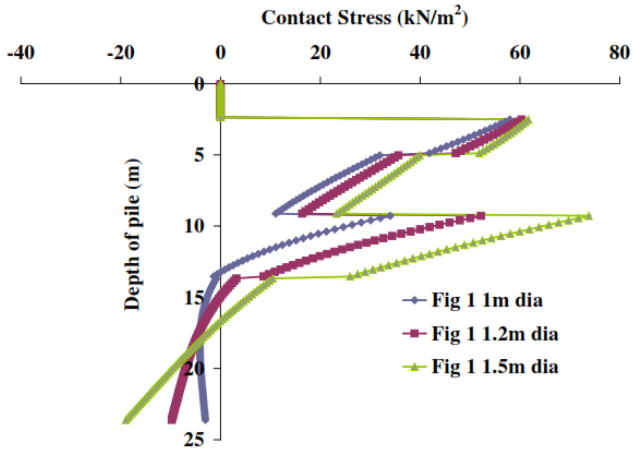

Fig 16d Effect of pile diameter on the contact stress profile of fixed headed pile (Problem 2)

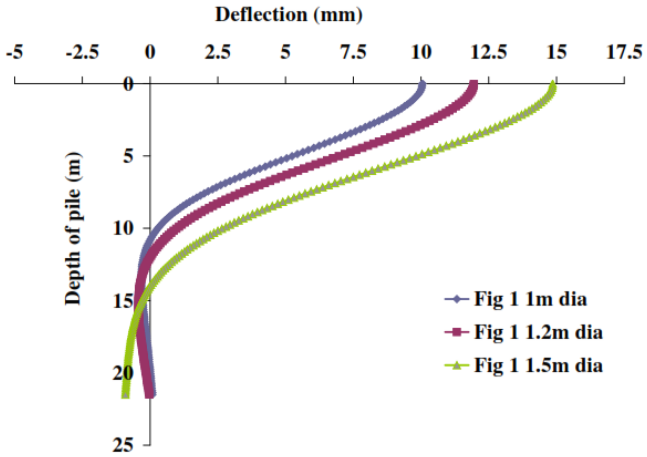

Fig 17a Effect of pile diameter on the deflection profile of fixed headed pile (Problem 3)

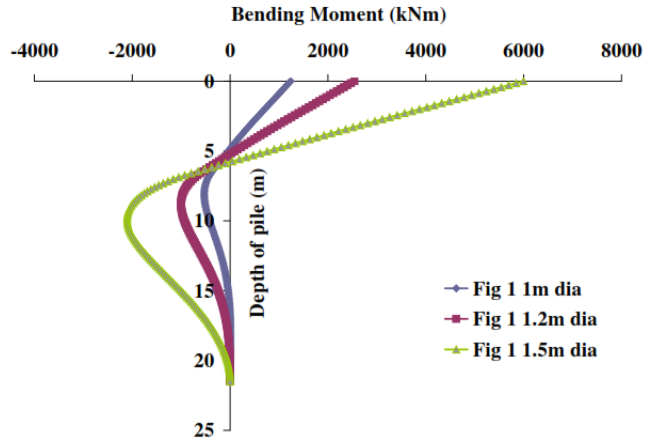

Fig 17b Effect of pile diameter on the bending moment profile of fixed headed pile (Problem 3)

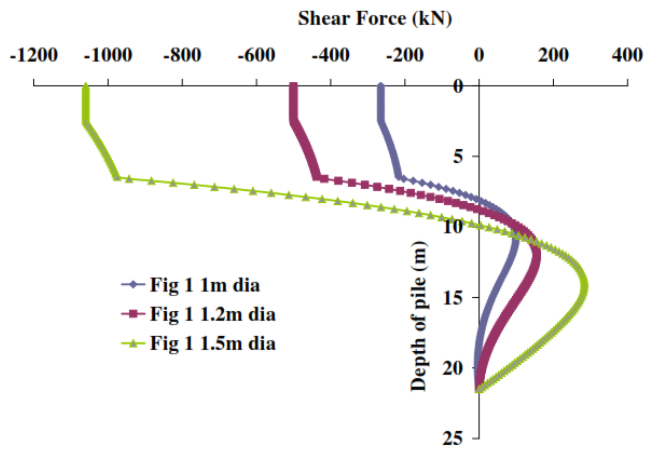

Fig 17c Effect of pile diameter on the shear force profile of fixed headed pile (Problem 3)

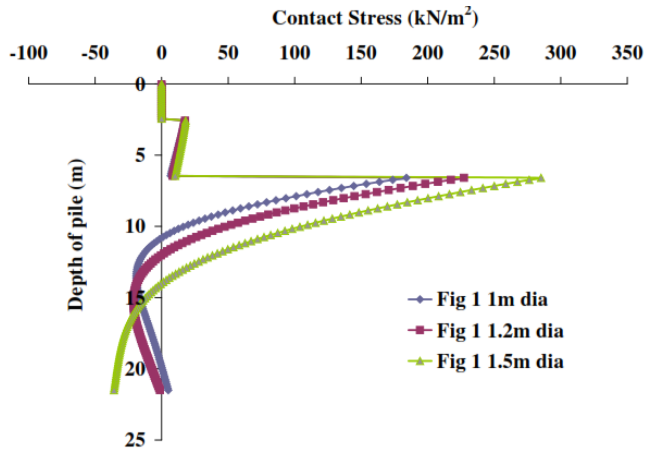

Fig 17d Effect of pile diameter on the contact stress profile of fixed headed pile (Problem 3)

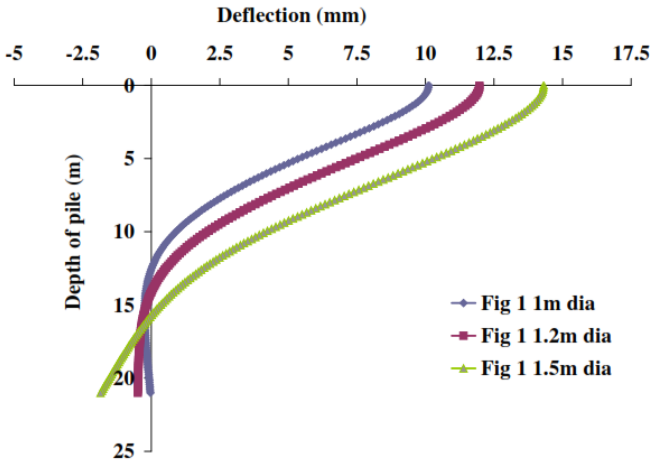

Fig 18a Effect of pile diameter on the deflection profile of fixed headed pile (Problem 4)

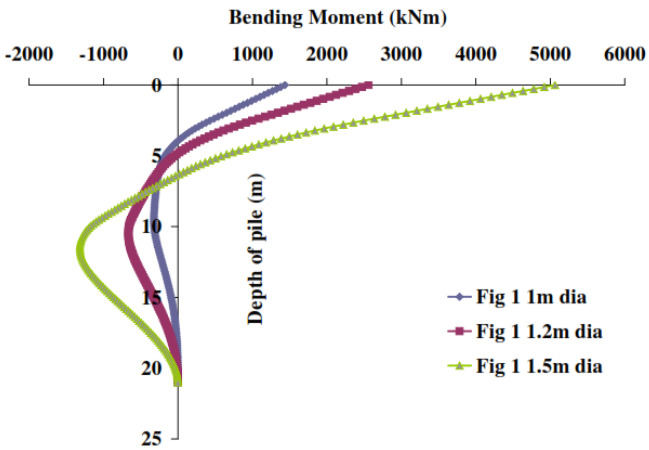

Fig 18b Effect of pile diameter on the bending moment profile of fixed headed pile (Problem 4)

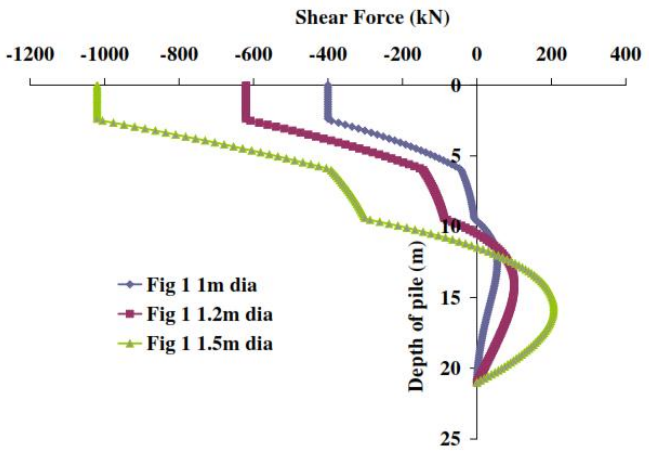

Fig 18c Effect of pile diameter on the shear force profile of fixed headed pile (Problem 4)

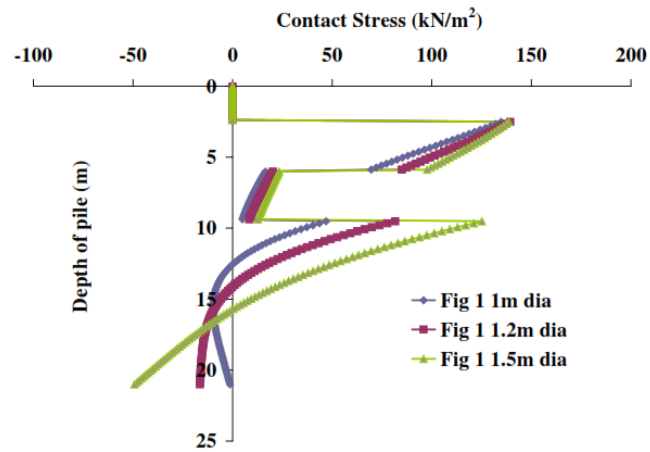

Fig 18d Effect of pile diameter on the contact stress profile of fixed headed pile (Problem 4)

Table 1 Response of the pile at lateral load capacity as obtained from BEF

Problem-1

D (mm)	$y_{cut\text{-}off}$ (mm)	P_u (kN)	M_{max} (kNm)	Q_{max} (kN)	p_{max} (kPa)	L_{fix} (m)
1000	10	420	1614.73	271.4	156.5	4.2
1200	12	700	3025.34	466.3	199.8	4.7
1500	15	1350	6737.04	947.9	260.9	5.5

Problem-2

D (mm)	$y_{cut\text{-}off}$ (mm)	P_u (kN)	M_{max} (kNm)	Q_{max} (kN)	p_{max} (kPa)	L_{fix} (m)
1000	10	240	1022.44	113.8	57.96	4.9
1200	12	385	1892.81	181.9	60.32	5.7
1500	15	660	3901.64	316.8	73.75	7.2

Problem-3

D (mm)	$y_{cut\text{-}off}$ (mm)	P_u (kN)	M_{max} (kNm)	Q_{max} (kN)	p_{max} (kPa)	L_{fix} (m)
1000	10	265	1237.54	231.4	184	4.8
1200	12	500	2541.59	452.0	227.3	5.12
1500	15	1060	5997.22	985.9	285.2	5.8

Problem-4

D (mm)	$y_{cut\text{-}off}$ (mm)	P_u (kN)	M_{max} (kNm)	Q_{max} (kN)	p_{max} (kPa)	L_{fix} (m)
1000	10	400	1436.93	217.8	134.8	4.0
1200	12	620	2558.87	270.9	139.4	4.7
1500	15	1020	5063.51	380.1	138.8	6.3

Table 1 provides the conglomerated estimations of the load carrying capacity of piles and their length of fixity for various diameters of piles embedded in different soil conditions. It can be noted that the enhancement in the pile diameter increases the lateral load capacity of the pile due to the increase in the stiffness of the pile. Moreover, the effective length of the pile (indicted by the location of point of fixity of the embedded pile) is also increased indicating that lesser length of pile would be participating in the soil-structure interaction as observed from the varying flexural response of the same.

6. CONCLUSIONS

Based on the conducted study, the following important conclusions can be stated:

- Mesh refinement aids in getting a stable and convergent solution. Mesh refinement in BEF revealed that 151 nodes proves to be sufficient to obtain a stable solution and can be considered as optimal number of nodes for the reported problems.
- BEF models the soil as a linear elastic material, and hence, the load-deformation behavior of the soil has also obtained to be linear.
- Owing to enhanced flexural rigidity, lateral load capacity of a pile increases with the increase in the diameter of the pile.
- An increase in the diameter of the embedded piles results in the increase in the maximums of the flexural responses of the pile.
- BEF results revealed that the length of fixity of the pile increased with the increase in the pile diameter.

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